A new family of power indices for voting games

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Abstract

In this paper we introduce a new family of power indices, especially designed for voting games, based on three parameters. A suitable choice of the parameters allows better taking into account particular features of each real political situation. We propose an archetype of the family and analyze some possible settings of the parameters. We make also some comparisons with existing indices.

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1 Introduction and motivation

Power indices are a relevant tool to measure the influence of each member of a coalition on decisions. In the last decade they have received increasing attention in political science, mostly because of the necessity both to study the voting power among EU member states and to analyze the effects of institutional reforms (e.g. Felsenthal and Machover, 1997; Nurmi, 1997 and 2000; Nurmi and Meskanen, 1999; Dowding, 2000; Aleskerov et al., 2002).

However, some features of power indices lead some scholars to call their usefulness in question. In particular, critiques are against the *a priori* nature of the power indices and on the fact that the assumption of random voting is far from real scenarios (Garrett and Tsebelis, 2001; Albert, 2003). Albert points out that the theory of power indices "should not [...] be considered as part of political science [... and cannot] be used for purposes of prediction or explanation" (p.1).

In defense of power indices, Leech (2002 and 2003) argues that the *a priori* power indices approach is not synonymous with the voting power approach in general. Proponents of power indices (e.g. Shapley and Shubik, 1954; Banzhaf, 1965, 1968; Coleman, 1971) are fully aware of the fact that *a priori* power indices are different from measures of empirical power. Obviously, their final goal should be different. "The former [should] enable us to analyze the properties of voting systems in purely constitutional terms [... and ...] to solve normative problems" (Leech, 2003, p.9), under the veil of ignorance. "On the other hand, the latter [should] rely on observed behavior and belongs to positive political science" (Leech, 2003, p.9). Taking account of this distinction, a more constructive approach suggests to extend the theory of power indices rather than abandon them (List, 2003). This route should reduce the distance between *a priori* power indices and measures of empirical power. Our approach belongs to this tradition.

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The aim of this paper is twofold. From the point of view of policy makers we want to provide a power index with predictive value. Therefore, we propose a scheme for designing a family of power indices that may be tailored on different situations with a suitable setting of some parameters. From a game theoretic point of view it is possible to choose the parameters in such a way that the null player axiom is no longer valid. The underling idea is to take into account the role of a party in favoring the formation of a majority, even if its influence on the majority in terms of seats (or percentage of votes) is null (see Moretti and Patrone, 2008 and related comments).

The paper is organized as follows. In Section 2 we present some basic elements of Game Theory and a brief description of some existing power indices. Section 3 deals with our archetype index. Section 4 is devoted to a discussion on *plausibility criterion* and on its relation with our index and some suitable refinements. In Section 5 we compare our index to some existing indices. Section 6 concludes.

2 Recall of Game Theory

A *cooperative game with transferable utility* (TU-game) is a pair (N, v), where $N = \{1, 2, ..., n\}$ denotes the *finite set of players* and $v : 2^N \to \mathbb{R}$ is the *characteristic function*, with $v(\emptyset) = 0$. $v(S)$ is the worth of coalition $S \subseteq N$, i.e. what players in S may obtain standing alone.

A game (N, v) is *simple* when $v : 2^N \to \{0, 1\}$, with $S \subseteq T \Rightarrow v(S) \leq v(T)$ [4] and $v(N) = 1$. If $v(S) = 0$ then S is a *losing* coalition, while if $v(S) = 1$ then S is a *winning* coalition. When all the proper subcoalitions $T \subset S$ are losing, S is a *minimal winning coalition*; when there exists at least one subcoalition $T \subset S$ that is losing, S is a *quasi-minimal winning coalition*. Given a winning coalition S, if $S \setminus \{i\}$ is losing then $i \in N$ is a *critical player* for S; if a player $i \in N$ is critical for no coalition then i is a *null player*.

A particular class of simple games is represented by the *weighted majority games*. The players are associated to a weight vector $w = (w_1, w_2, ..., w_n)$ that leads to the following definition of the characteristic function of the corresponding weighted majority game (N, w):

$$w(S) = \begin{cases} 1 & \text{if } \sum_{i \in S} w_i > q \\ 0 & \text{otherwise} \end{cases}$$

where q is the *majority quota*. A weighted majority situation is often denoted as $[q; w_1, w_2, ..., w_n]$. Usually we ask that the game is *proper* or *N-proper*, i.e. that if S is winning then $N \setminus S$ is losing; for this aim it is sufficient to choose $q \geq \sum_{i \in N} w_i$.

An *allocation* is a n-dimensional vector $(x_i)_{i \in N} \in \mathbb{R}^N$ assigning to player $i \in N$ the amount x_i; an allocation $(x_i)_{i \in N}$ is *efficient* if $x(N) = \sum_{i \in N} x_i = v(N)$. A *solution rule* is a function ψ that assigns an allocation $\psi(v)$ to every TU-game belonging to a given class of games $\mathcal{G}$ with player set N.

For simple games, and in particular for weighted majority games, a solution is often called a *power index*, as each component x_i may be interpreted as the power assigned to player $i \in N$. In the literature, several power indices were introduced.

[4] This property is called *monotonicity*.

The *Shapley-Shubik index* (Shapley and Shubik, 1954), ϕ, is the natural extension of the Shapley value (Shapley, 1953) to simple games. Consider the set Π of all the orderings (permutations) of the players and for each ordering $\pi \in \Pi$ let $P(i, \pi)$ be the set of players that precede player i in the ordering π; the Shapley value is the average marginal contribution of each player w.r.t. all the possible orderings:

$$\phi_i(v) = \frac{1}{n!} \sum_{\pi \in \Pi} \left[v(P(i, \pi)) \cup \{i\}) - v(P(i, \pi)) \right], \ i \in N$$

where n denotes the cardinality of the set of players N.

The normalized *Banzhaf-Coleman index* (Banzhaf, 1965 and Coleman, 1971), β, is similar to the Shapley-Shubik index, but it considers all the marginal contributions of a player to all possible coalitions, without considering the order of the players. Let us introduce:

$$\beta_i^*(v) = \frac{1}{2^{n-1}} \sum_{S \subseteq N, S \ni i} \left[v(S) - v(S \setminus \{i\}) \right], \ i \in N$$

where again n denotes the cardinality of the set of players N.

By normalization we get:

$$\beta_i(v) = \frac{\beta_i^*(v)}{\sum_{j \in N} \beta_j^*(v)}, \ i \in N$$

The *Deegan-Packel index* (Deegan and Packel, 1978), δ, considers only the minimal winning coalitions; the power is firstly equally divided among them and then the power of each is equally divided among its members. Let $\mathcal{W} = \{S_1, S_2, ..., S_m\}$ be the set of minimal winning coalitions, formally:

$$\delta_i(v) = \sum_{S_k \in \mathcal{W}; S_k \ni i} \frac{1}{m} \frac{1}{s_k}, \ i \in N$$

where m and c_k denote the cardinalities of the set of minimal winning coalitions W and of the coalition S_k, respectively.

The *Johnston index* (Johnston, 1978), γ, considers only the quasi-minimal winning coalitions; the power is firstly equally divided among them and then the power of each is equally divided among its critical players. Let $\mathcal{W}^q = \{S_1, S_2, ..., S_q\}$ be the set of quasi-minimal winning coalitions and let $\mathcal{W}_i^q = \{S_1, ..., S_{\ell_i}\}$ be the set of quasi-minimal winning coalitions in which player i is critical, formally:

$$\gamma_i(v) = \sum_{S_k \in \mathcal{W}_i^q} \frac{1}{q} \frac{1}{c_{S_k}}, \ i \in N$$

where q denotes the cardinalities of the set of quasi-minimal winning coalitions $\mathcal{W}^q$ and c_{S_k} the number of critical players of coalition S_k.

The *Holler index* (Holler, 1982), h, or *Public Goods index* is defined as follows; first we consider the number c_i of minimal winning coalitions which player $i \in N$ belongs to, then, by normalization, we get:

$$h_i(v) = \frac{c_i}{\sum_{j \in N} c_j}, \ i \in N$$

3 The Archetype

In this section we introduce the basic characteristics of our family of power indices.

Let us consider a *set of parties* $N = \{1, 2, ..., n\}$ and a *vector of weights* $w = (w_1, w_2, ..., w_n)$, that may be viewed as percentages of votes, number of seats or other measures of the "weight" of the players [5]. Fixing a suitable majority quota q, we obtain a weighted majority situation $[q; w_1, w_2, ..., w_n]$ and the associated weighted majority game (N, w) that is monotonic and proper.

We may represent the parties on a left-right axis, via a suitable analysis of their ideologies, and we suppose that the players are ordered according to their position on this axis. Starting from this point, we may consider the set of winning coalitions, with the hypothesis that the parties are contiguous, i.e. given a winning coalition S, for all $i, j \in S$ if there exists $k \in N$ with $i < k < j$ then $k \in S$.

Formally, let $\mathcal{W}^c = \{S_1, S_2, ..., S_m\}$ be the set of winning coalitions with contiguous players; in the first step the unitary power is equally shared among coalitions $S_j \in \mathcal{W}^c$ and in the second step the quota assigned to each coalition S_j is equally shared among its members. Finally, each player $i \in N$ sums up all the amounts received in the coalitions he belongs to.

Denoting the power index by FP (after Family of Power indices) we have:

$$FP_i(v) = \sum_{S_k \in \mathcal{W}^c; S_k \ni i} \frac{1}{m} \frac{1}{s_k}, \ i \in N$$

where m denotes the cardinality of the set $\mathcal{W}^c$ and s_j denotes the number of players in coalition S_j.

In the following we generalize the definition allowing a wider meaning of the elements involved. More precisely, we consider (i) the possibility of a subset of contiguous winning coalitions, (ii) the sharing of the unitary power among them according to a probability distribution and (iii) the sharing of the power inside a coalition taking into account possible differences among the parties.

4 The Plausibility Criterion

The traditional power indices are charged to be far from measures of empirical power. The main point is that they are based on the assumption of random voting that prevents from any distinction between logically possible coalitions and feasible ones. The exclusion of political preferences also implies that, once a set of winning coalitions is defined, each of them has the same probability to form.

Our answer to this approach is what we call *plausibility criterion.* The idea is that a power index with predictive value should be computed on the basis of plausible scenarios - the definition of the set of winning coalitions, of the probability a coalition forms and the share of power within each coalition should refer to political and

[5] We are aware of the fact that both the electoral system and pre-electoral alliances may change the distribution of the votes among parties. In this work, we assume that the Parliament has already formed but the government hasn't been defined yet. This allows us to analyse a post-strategic-voting scenario.

historical issues. The nature of our FP index allows respecting the plausibility criterion by changing three parameters in a suitable way - the set of winning coalitions, the probability of a winning coalition to form, the sharing rule inside each winning coalition.

4.1 The set of winning coalitions

Coalition formation is a key issue in political life. Consequently, this topic has been studied by several scholars who tried and answered the question: What is the mechanism through which coalitions form? The result is that the literature is now replete with theoretical models that explain why some coalitions form while others do not.

The *office-seeking* approach assumes that the size of the coalition is the only relevant feature that determines the plausibility of the existence of a coalition. The models referring to this tradition are based on the idea that parties are only interested in the advantages related to the office. Consequently, only minimal coalitions are plausible. Three types of minimality are considered in the office-seeking literature. The minimal winning coalition (*MWC*) hypothesis, where none of the partners is mathematically superfluous (von Neumann and Morgenstern, 1953). The minimal number coalition (*MNC*) hypothesis, where a government is formed by the minimum number of parties to avoid negotiation costs (Leiserson, 1966). The minimal size coalition (*MSC*) hypothesis where only coalitions which obtain the smallest possible majority of seats form (Riker, 1962).

This apolitical approach results into some questionable outcomes. First of all, the ideological position of each party is irrelevant. According to these models, a coalition between far-left and far-right parties may emerge if the minimality condition is met. Then, non-minimal coalitions are definitely banned. Finally, these indices assign a null power to null players, simply on the basis of the characteristic function but despite the possible role of catalyst they may play in a majority formation process.

The *policy-seeking* approach tries and provides an improvement of power indices in this direction. Axelrod's intuition (1970) is that coalitions form if they are ideologically connected along a policy dimension. In other words, parties in a coalition should have adjacent preferred positions on a uni-dimensional ideological line. De Swaan (1973) provides a refinement of Axelrod's theory by suggesting that not only the ordering but also the exact position matters. Parties take account of the distance between the policy program of a potential government and their preferences when deciding whether to participate or not in a coalition. Consequently, only coalitions with the smallest ideological range emerge. Aleskerov (2008) proposes a more sophisticated procedure - a consistency index - to select realistic winning coalitions on the basis of the ideological distance, even if the basic idea is again to consider only ideologically close coalitions.

We appreciate the policy-seeking approach much more than the office-seeking one. The reason is that no plausible coalition may be studied if ideological issues are left apart. This would imply, for instance, that non-minimal coalitions are banned even if they could logically contribute to fulfilling the connectedness criterion (Bäck, 2001) and to augment the opportunity to carry out a particular political program. Another reason to deserve a relevant role to non-minimal coalitions is that they may be more stable. In fact, adding a party to a minimal winning coalition implies that small parties are less likely to blackmail the big ones. Italian political tradition, for instance, offers a relevant number of scenarios where non-minimal governments have formed. The most striking example is from 1948 to 1992 where in all governments Christian Democrats - who obtained the absolute majority of seats - headed non-minimal governments. This suggests the possibility of selecting only the winning coalitions that include the relative majority party.

Our FP belongs to the policy-seeking tradition, even if we leave several degrees of freedom on the definition of winning coalitions. In fact, the only essential requirement is contiguity of parties, while the procedure to define the winning coalitions may be decided on the basis of the political situation we want to study.

4.2 The probability of a coalition to form

As we argued before, the plausibility criterion implies that a suitable analysis of coalition formation cannot ignore political and historical issues. Consequently, equiprobability of coalitions may be not an appealing feature of a power index - even if our FP allows for that option.

Spatial models approach suggests several techniques to determine the probability of the formation of coalitions. Bilal et al. (2001) propose two alternatives. A first possibility is to assume that the probability a coalition forms is a function of the distance between the extreme parties. The other option is to assume that the shorter the distance between the preferred point of each party and the position of the coalition the more likely the agreement is reached.

We may also mention two possible methods for assigning the probability of formation of a majority. The first is based on the number of parties. It can be viewed as a proxy of the ideological distance of the extreme parties - the more the parties, the larger the distance - if we suppose that the parties are located at equal distance on the left-right axis. The second considers the number of parties or the number of seats as a cost of coordination and agreement - the larger the cost, the lower the probability.

A more applied approach is suggested by Heard and Swartz (1998). They provide an empirical analysis where they derive the probability of a coalition to form through estimation on historical voting.

Generally speaking, our FP allows also in this case the use of several different procedures according to the political scenario under analysis.

4.3 The sharing rule inside each winning coalition

In line with the previous discussion, the plausibility criterion allows to say something also about the sharing rule inside each winning coalition. In particular, it suggests that the equal sharing rule may be not a suitable feature - even if, also in this case, our index does not exclude this option.

A possible cumbersome option is to allocate the power according to the number of seats each party obtains. Empirical evidence supports this solution. In Italy, for instance, the number of ministers each party is assigned is typically correlated to the number of seats obtained. Moreover, the premier is usually the leader of the major party.

Another feature to take into account is the critical role played by parties - the faculty of making a government fall. When a party is “critical” for the existence of a coalition, it is endowed with power. Therefore, it may be realistic to weight power on the basis of its influence on the coalition life. A more sophisticated refinement of this assumption may assign more power to critical parties that are closer to the potential opposition. This is due to the fact that they are more likely to leave the coalition and to join the opposition as some internal crisis occurs. Another input comes from the median voter theorem (Hotelling, 1929; Downs, 1957). Under the assumption of unidimensional policies and single-peakedness of voters' utility functions, the policy preferred by the median voter is the only one preferred by the majority of the voters. This implies a high bargaining power of the median party within the coalition (Bäck, 2001).

5 Examples and Comparison with Existing Power Indices

In this section we compare our index with other classical indices. First of all, we want to remark that the FP index results to be far from indices like Shapley-Shubik and Banzhaf-Coleman that are based on the marginal contribution of players to all possible coalitions. Moreover, we may add, in a general framework, that also other indices like Deegan-Packel, Johnston and Holler that are more similar to our one because they are rooted in the sharing of power among the members of some winning coalitions may give different results. For instance, these indices assign a null power to null players that are excluded from the sharing mechanism.

Other comparisons may be carried out starting from the following examples.

Example 1 *Let* $[9; 6, 2, 6]$ *be a weighted majority situation. In this case we have a unique contiguous winning coalition* $\{1, 2, 3\}$*. The equal sharing of the power among the players provides* $FP = (0.333, 0.333, 0.333)$*. If we compute the indices* $\phi, \beta, \delta, \gamma$ *and* h *we obtain* $(0.5, 0, 0.5)$*.* $\diamondsuit$

The different power assigned to the parties depends on the role of the second party that on the one hand has the same role of the others in a contiguous winning coalition, and on the other hand is a null player.

The following example shows how different sharing rule may better take into account some characteristics of the situation we are facing.

Example 2 *Let* $[6; 1, 5, 2, 3]$ *be a weighted majority situation. In this case we have four contiguous winning coalitions* $\{2, 3\}, \{1, 2, 3\}, \{2, 3, 4\}, \{1, 2, 3, 4\}$*. The archetype index is* $FP = (0.146, 0.354, 0.354, 0.146)$*, whereas* $\phi = (0, 0.666, 0.167, 0.167), \beta = (0, 0.6, 0.2, 0.2), \delta = h = (0, 0.5, 0.25, 0.25)$ *and* $\gamma = (0, 0.583, 0.250, 0.167)$*.* $\diamondsuit$

The main difference is that the first party obtains always, but for FP, a null power. If we select a subset of contiguous winning coalitions, namely the minimal ones, we have only coalition $\{2, 3\}$, so $FP = (0, 0.5, 0.5, 0)$. If we share the power among the members of each coalition proportionally to the number of seats we have $FP = (0.054, 0.573, 0.229, 0.143)$. If we modify the sharing rule among the coalitions on the basis of the number of parties, i.e. assigning the probability equal to the normalized inverse of the number of members $(0.353, 0.235, 0.235, 0.176)$, maintaining the proportional sharing w.r.t. the seats, we have $FP = (0.045, 0.597, 0.239, 0.119)$. In this way we increase the power of parties 2 and 3. In the following example we use a sharing rule inside a winning coalition based on a different definition of critical player, i.e. a player $i \in S$ is *contiguous-critical* for a contiguous winning coalition S if $S \setminus \{i\}$ is losing or is no longer contiguous.

Example 3 *Let* $[24; 13, 8, 2, 10, 11]$ *be a weighted majority situation. In this case we have three contiguous winning coalitions* $\{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{1, 2, 3, 4, 5\}$. *In the first two coalitions all parties are contiguous-critical, while in the third one only parties* $2, 3, 4$ *are contiguous-critical. So, the sharing vectors are* $(0.25, 0.25, 0.25, 0.25), (0.25, 0.25, 0.25, 0.25), (0, 0.333, 0.333, 0.333, 0)$, *respectively. The resulting index is* $FP = (0.083, 0.277, 0.277, 0.277, 0.083)$. $\diamondsuit$

6 Concluding Remarks

In this paper we provide a new family of power indices. The peculiarity of this family is that it may be tailored on different situations with a suitable setting of some parameters. This allows studying different political scenarios in a more realistic way. In the fourth section of the paper we suggest some relevant issues that may be taken into account when setting parameters. Potentially, the combination of all possible criteria to set parameters leads to a high number of indices. However, both internal and external validity of the index should be observed. The former requires that the assumptions concerning the way parameters are chosen should be coherent. The latter entails a set of parameters as close as possible to the real one. Empirical analyses on real data and estimation of the parameters may play a relevant role.

About the coherence in the choice of the parameters we may mention that a sharing rule among the member of a coalition based on the number of seats is suitable in case of minimal size coalition hypothesis; analogously a sharing rule among the winning coalitions based on the number of parties is suitable in case of minimal number coalition hypothesis. On the other hand, it would be inconsistent if a larger number of parties - or seats - implied a reduced probability for a coalition to form.

Starting from our FP several developments are possible. For example, the analysis of the selection rules for the parameters in order to obtain classical power indices deserves further inquiry. Of course this result would be straightforward for a fixed voting situation. Therefore, the aim should be at least to identify classes of voting games for which a suitable choice of the parameters allows to make the FP index equal to another index.

Another interesting issue is multidimensionality. At the moment our FP deals only with unidimensional policies. A multidimensional approach would allow taking account of the multifaceted nature of voters' decisions.

Endogeneity due to strategic behavior - of both voters and parties - could be another relevant topic. In this paper we analyze power distribution and coalition formation as if they could not influence voters' decisions. However, it is possible that the coalition a party will join to can influence the votes it receives and, consequently, its power. As a result, parties may decide to enter a coalition only if it maximizes their power.

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