# Aspects of Power Overlooked by Power Indices*

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**Abstract**

The *a priori* voting power indices concentrate on actor resource distributions and decision rules to determine the potential influence over outcomes by various actors. That these indices sometimes seem to be at odds with the intuitive distribution of real power in voting bodies follows naturally from their *a priori* nature. Indices based on actor preferences address this by equating an actor's voting power with the proximity of voting outcomes to his/her ideal point. It is, however, shown that in some cases the preference-based indices are just as questionable as the classic ones. The main aim of this paper is to delineate the proper scope of power indices. In the pursuit of this aim we try to show that the procedures resorted to in making collective decisions are as important – if not more so – as the actor resource distribution. We review some results on agenda-systems to drive home this point. The proper role of power indices then turns out to be in the study of actor influences over outcomes when the actors are on the same level of aggregation (individuals, groups, states) and "comparable" in the sense of having similar sets of strategies at their disposal and preferences are not taken into consideration, *e.g.* because a veil of ignorance applies.

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# 1 Introduction

The background of voting power indices is in game theory and measurement theory. One of their uses is to provide an estimate for the value or payoff that an actor may expect to receive when entering a game. As such they are akin to means, modes and medians. The typical setting of power indices involves evaluation: from a given resource (vote) distribution one aims to estimate the actors' influence over decision outcomes when basically nothing is known about the issues to be decided upon in the game. For example, proportional representation (PR) systems aim at a distribution of parliamentary seats among parties that is nearly identical with the distribution of support given to those parties in the elections. Thereby an (implicit) assumption is made that distribution of seats coincides with the distribution of legislative influence. This assumption is, however, untenable: x% of seats does not in general give a party x% control over legislation: given majority voting, 51% of seats imply full control. The *a priori* voting power indices aim to rectify this by explicitly introducing the decision rule so that it is the decision rule together with the vote distribution that determines the influence over outcomes.

By taking into account the decision rules, the standard *a priori* voting power indices take a step towards measuring the influence of actors on the decision outcomes. The fact that they sometimes deviate from independent observations about power distribution can partly be explained by their very *a priori* nature. For example, if an index "assumes" that all coalitions of actors are equally likely, it is to be expected that it provides poor estimates of power distribution in bodies where large classes of coalitions are impossible or extremely rare because of ideological constraints.

The classical power indices have for some time been criticized for ignoring the preferences of actors in coalition formation. In response to this criticism a new type of indices – often called preference-based ones – has been developed (Steunenberg et al. 1999; Napel and Widgrén 2005, 2009). In those indices the power is measured in terms of the distance of outcomes to the actors' ideal points. The main issue in this paper is that the standard *a priori* voting power indices do what they are supposed to do under very special circumstances only. The same is true – albeit for different reasons – of the preference-based indices.

In the next section the classical indices are introduced and briefly motivated. It is standard to relate them to yes-no decision making. However, in following section 3 it is shown that dichotomous voting typically takes place in a multi-alternative environment, i.e. while the vote is taken between two alternatives at each stage of the procedure, there are several interdependent binary votes in the process. The agenda determines the sequence of these votes. Under certain types of behavioral assumptions the sequence also crucially restricts the feasible outcomes. It is argued that, when compared with marginal changes in voter resource distribution, the control of agenda is of essentially greater importance with regard to the voting outcomes.

Section 4 deals with various monotonicity-related paradoxes in an effort to demonstrate that power under some widely used voting procedures in multiple-alternative settings is not locally monotonic. Hence indices based on this type of monotonicity fail to capture the distribution of power under those procedures. In section 5 we deal with the issue of how voting procedures influence the voting power distribution and whether preference proximity considerations are reconcilable with other intuitively plausible choice principles. In section 6 we turn to paradoxes of composition to illustrate how the very notion of proximity may become ambiguous even in simple game, i.e. dichotomous settings.

## 2 A priori power indices

The Shapley-Shubik power index (S-S) is a projection of the Shapley value to simple games (Shapley 1953; Shapley and Shubik 1954).[1] It can viewed as a measure based on the assumption that all attitude dimensions (sequences of decision makers in order from the most supportive to the least supportive one) are equiprobable. The two indices named after Penrose and Banzhaf replace this equiprobability of dimensions assumption with one that pertains to actor coalitions (Penrose 1946; Banzhaf 1965). The standardized Penrose-Banzhaf (P-B) index counts for each player the number of winning coalitions where this player has a swing, *i.e.* where his presence is, *ceteris paribus*, crucial for the coalition to be winning, and divides this number by the sum of swings of all players. The absolute Penrose-Banzhaf index counts the number of swings and divides this by the number of coalitions where the player is present. In contrast to the previous ones, the values of absolute Penrose-Banzhaf index, when summed over the actors, do not in general add up to unity.

In all these three indices the power of a player is determined by the number of winning coalitions in which he is present as an essential member in the sense that should he leave the coalition, it would become non-winning.

Two more recent indices, viz. the public good index(PGI), introduced in Holler (1982), and the Deegan-Packel (1982) index, focus on the minimal winning coalitions, i.e. on coalitions in which all members are decisive in the sense that should *any* one of them leave the coalition, it would become non-winning. The importance of players, and consequently their payoff expectation, is according to the designers of these indices reflected by the number of presences in these types of coalitions.

Table 1 illustrates the above indices in the now bygone EU-15. The differences between the Shapley-Shubik and standardized Penrose-Banzhaf

[1] The computation formulae of the indices are listed in Appendix.

| *country* | *no. of votes* | *S-S index* | *std P-B index* | *DP index* | *Holler index* |
|---|---|---|---|---|---|
| F, G, I, UK | 10 | .1167 | .1116 | .0822 | .0809 |
| S | 8 | .0955 | .0924 | .0751 | .0743 |
| B, G, N, P | 5 | .0552 | .0587 | .0647 | .0650 |
| A, S | 4 | .0454 | .0479 | .0608 | .0613 |
| D, Fi, Ir | 3 | .0353 | .0359 | .0572 | .0582 |
| L | 2 | .0207 | .0226 | .0440 | .0450 |

Table 1: The Shapley-Shubik (S-S), standardized Penrose-Banzhaf (P-B), Deegan-Packel (DP) and PGI Values of Countries in the EU-15 for the Rule 62/87.

index values are in general very small. The same observation holds for the two indices based on swings in minimal winning coalitions: DP and PGI. Note that countries with larger voting weights have at least as large power values as countries with smaller voting weights.

This monotonicity property, *i.e.* local monotonicity, is not always satisfied for the values of the Deegan-Packel index and the PGI. The following voting game illustrates this. A voting body consists of 6 persons with voting weights $3, 3, 1, 1, 1, 1$. The decision rule is 6, *i.e.* any coalition with the sum of voting weights of at least 6 is winning. This yields the PGI value distribution: $5/34, 5/34, 6/34, 6/34, 6/34, 6/34$. In other words, the players with larger voting weights have a smaller PGI value than those with smaller weights. Hence, local monotonicity is violated.

## 3 Agenda-based procedures

The simple games are the domain of the above indices of *a priori* voting power. There are circumstances where simple games are quite natural analysis devices. For example, the votes of confidence or non-confidence in parliamentary systems would seem like simple games in requiring the voters (MPs) to choose one of two exhaustive and mutually exclusive alternatives. Similarly, in most parliaments legislative outcomes are determined on the basis of a binary vote where the winning alternative defeats its competitor in the final contest. Upon closer scrutiny, however, most legislative processes involve more than two decision alternatives. In committee decisions the agenda-building is typically preceded by a discussion in the course of which various parties make proposals for the policy to be taken or candidates for offices. By agenda-based procedures one usually refers to committee procedures where the agenda is explicitly decided upon after the decision alternatives are known. Typical settings of agenda-based procedures are parliaments and committees. One of the crucial determinants of voting power overlooked by power indices is the power of agenda-builder.

Two procedures stand out among the agenda-base systems: (i) the amendment and (ii) the successive procedure. Both are widely used in contemporary parliaments. The successive one is based on pairwise comparisons. At each stage of this procedure an alternative is confronted with the set of all remaining alternatives. If it is voted upon by a majority, it is elected and the process is terminated. Otherwise this alternative is set aside and the next one is confronted with all the remaining alternatives. Again the majority decides whether this alternative is elected and the process terminated or whether the next alternative is picked up for the next vote. Eventually one alternative gets the majority support and is elected.

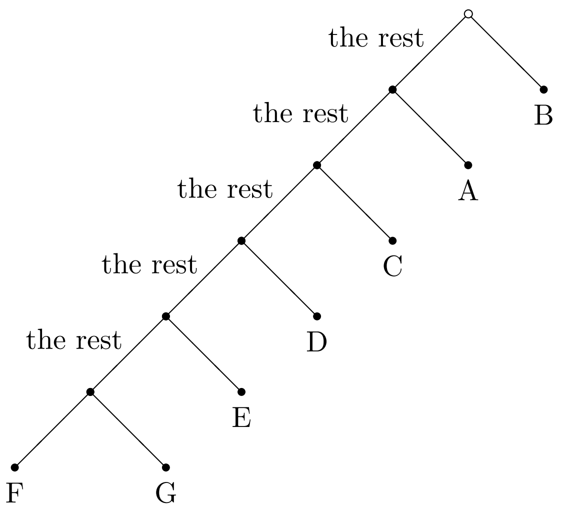


Figure 1: The successive agenda

Figure 1 one shows an example of a successive agenda where the order of alternatives to be voted upon is B, A, C, D, E, F and G. Whether this sequence will be followed through depends on the outcomes of the ballots. In general, the maximum number of ballots taken of $k$ alternatives is $k-1$. If an alernative gets a simple majority of votes, it is selected as winner.

The amendment procedure confronts alternatives with each other in pairs so that in each ballot two separate alternatives are compared. Whichever gets the majority of votes proceeds to the next ballot, while the loser is set aside. Figure 2 shows and example of an amendment agenda over 3 alternatives: A, B and C.

In Figure 2 alternatives A and B are first compared and the winner is faced with C on the second ballot.

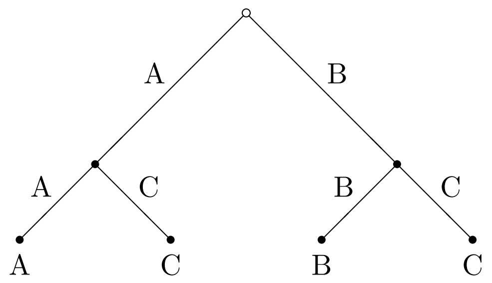


Figure 2: The amendment agenda

Both the amendment and successive procedure are very agenda-sensitive systems. In other words, two agendas may produce different outcomes even though the underlying preference ranking of voters and their voting behavior remain the same. Under sincere voting – whereby, for all pairs of alternatives A and B, the voter always votes for A if he prefers A to B and vice versa – the well-known Condorcet's paradox provides an example: of the three alternatives any one can be rendered the winner depending on the agenda. To determine the outcomes – even under sincere voting – of successive procedure requires additional assumptions regarding voter preferences over subsets of alternatives. If the voters always vote for the subset of alternatives that contains their first-ranked alternative, the successive procedure is also very vulnerable to agenda-manipulation.

The agenda-based systems have received some attention in the social choice theory. Thus, we know e.g. the following about the amendment and successive systems:

1. Condorcet losers are not elected (not even under sincere voting).
2. Sophisticated voting avoids the worst possible outcomes, i.e those outside the Pareto set.
3. The Condorcet winner is elected (both under sincere and strategic voting) by the amendment procedure.
4. The strong Condorcet winner is elected by both systems.

The first point follows from the observation that the alternative that wins under the amendment procedure has to beat at least one other alternative. Hence, it cannot be the Condorcet loser either. Under the successive procedure if the winner is determined at the final pairwise vote, it cannot be the Condorcet loser. If, on the other hand, the winner appears earlier, it cannot be the Condorcet loser either because it is ranked first by more than half of the voting body.

Sophisticated voting avoids Pareto violations. In other words, if the voters anticipate the outcomes ensuing from various voting strategies, the resulting strategy combinations exclude outcomes for which unanimously preferred outcomes exist (see Miller 1995, 87).

That the amendment procedure results in the Condorcet winner under sincere voting, follows from the definition. Finally, the strong Condorcet winner – i.e. one that is ranked first by more than half of the electorate – is elected by both systems regardless of whether the voting is sincere or strategic.

To counterbalance the basically positive results mentioned above, there are some negative ones such as,

1. McKelvey's (1979) results on majority rule and agenda-control.
2. All Condorcet extensions are vulnerable to the no-show paradox (Moulin 1988).
3. Pareto violations are possible under sincere voting.

McKelvey's well-known theorem states that under fairly general conditions – multi-dimensional policy spaces, continuous utilities over the policy space, empty core – *any* alternative can become the voting outcome under amendment procedure if the voters are sincere and myopic. Under these circumstances the agenda-controller determines the outcome even though at every stage of voting the majority determines the winner of the pairwise vote. Although some of the conditions are not so liberal as they seem at first sight, the theorem is certainly important in calling attention to the limits – or rather, lack thereof – that the majority rule *per se* can impose on the possible outcomes. The upshot is that the majority rule guarantees no correspondence between voter opinions and voting outcomes.

Although no analogous result on the outcomes of the successive procedure in multi-dimensional policy spaces exists, it also can be shown to be very vulnerable to agenda-manipulation (Nurmi 2010). In conclusion, then, ignoring the process whereby the sequence of pairwise votes is determined can result in a misleading picture of the influence that various actors exert upon the decision outcomes. Admittedly, the power of the agenda-builder can to some extent be counteracted through sophisticated voting, but even so the best – and in itself exhaustive – characterization of the outcomes reachable by pairwise majority voting, i.e. the Banks set, sometimes leaves a significant maneuvering room for the agenda-builder.

Local monotonicity is a property that many scholars deem particularly important. What it states is that increasing an actor's resources (votes, shares of stock), *ceteris paribus*, is never accompanied with a diminution of his voting power. It is known that the Shapley-Shubik and the Penrose-Banhaf indices are locally monotonic, while the indices based on minimal winning coalitions, the Deegan-Packel index and PGI, are not. But is the influence over outcomes always locally monotonic?

## 4 More votes, less power

The intuitive view of power – voting power included – is based on two tenets:

- the more resources an actor controls, the more often he is on the winning side
- the more powerful an actor, the closer his preferences are to the collective decisions.

Let us look at the former claim first. In voting studies, the resources are typically votes in a voting body. The tenet, thus, has it that the more votes, the more powerful the decision maker. In situations involving more than two alternatives, this tenet has to be essentially qualified, if not downright rejected on the grounds that some widely used voting rules contradict it. In other words, the tenet is at least not universally applicable. In fact, two social choice properties are directly relevant for the rejection of the tenet: non-monotonicity and vulnerability to the no-show paradox. The former means that under some preference profiles it is possible that additional support, *ceteris paribus* would render a winning alternative a non-winning one. On the other hand, systems where some voters might end up with more preferable outcomes by not voting at all than by voting according to their preferences, are vulnerable to the no-show paradox. These two properties are closely related, but not equivalent.

Table 2 illustrates the non-monotonicity of plurality runoff system. Assuming that everyone votes according his preference, i.e. the voting is sincere, the plurality runoff results in A. Suppose now that the winner had had somewhat more support so that two of the voters with $B \succ C \succ A$ ranking had lifted A first, *ceteris paribus*. In this new profile, the runoff would take place between A and C, whereupon C would win. Hence, clearly the $A \succ B \succ C$ group would have done better – been more powerful – with less votes.[2]

Table 3 illustrates a related phenomenon. By abstaining a group of voters may – *ceteris paribus* – improve upon the outcome that would result if they voted according to their preferences. The example is again based on plurality runoff system. With sincere voting, A wins, but if two voters in the $B \succ C \succ A$ group abstain, C wins, an improvement upon A from the

[2] A referee correctly points out that the outcomes under sincere voting are not Nash equilibria. Indeed, none of the three outcomes is a Nash equilibrium. To wit, if A is the outcome, then Group 2 has an incentive to vote for C at the outset making it thereby the strong Condocet winner and hence the plurality runoff winner as well. The same argument applies *mutatis mutandis* to the two other outcomes B and C.

| *22 voters* | *21 voters* | *20 voters* |
|---|---|---|
| A | B | C |
| B | C | A |
| C | A | B |

Table 2: Additional support paradox

| 5 voters | 5 voters | 4 voters |
|---|---|---|
| A | B | C |
| B | C | A |
| C | A | B |

Table 3: No-Show Paradox

view-point of the abstainers. Provided that C is closer to the abstainers' preferences than A, the second tenet above is again contradicted.[3]

One more argument can be presented in contradiction to the above tenets. Schwartz (1995) calls it the paradox of representation. But since there are several paradoxes related to representation we shall call it Schwartz' paradox. It is useful to illustrate it in terms of the amendment procedure. Consider Table 4.

Suppose that in parliamentary debate a motion $b$ has been presented and that also an amendment to it $c$ is on the table. Hence we have the amendment agenda:

- motion $b$ vs. amendment $c$,
- the winner of the preceding vs. $a$

With sincere voting $a$ emerges as the winner. Suppose now that party B would lose all its seats so that parties A and C would share those seats equally. Thus, $c$ would become the (strong) Condorcet winner and hence the winner of the contest here. Again clearly a violation of the tenets above.

The above examples are procedure-related and thus basically avoidable by choosing a monotonic voting system, such as plurality voting or Borda count.

[3] We shall here deal with the general no-show paradox only and omit its strong version. A more comprehensive account of both types is given in Nurmi (2011) which is to a large extent a result of private correspondence with Dan S. Felsenthal dating back to May 2001 and continuing intermittently till early 2011.

| party A<br>23 seats | party B<br>28 seats | party C<br>49 seats |
|---|---|---|
| a | b | c |
| b | c | a |
| c | a | b |

Table 4: Schwartz' Paradox: An Example

## 5 Power and preference proximity

Consider a voting body and a very small group of voters with identical preferences in it. Suppose that the voters make a mistake in reporting their preferences in an election. One of the group members may have interpreted the content of decision alternatives incorrectly and the others are following his lead in reporting their preferences in voting. Since we are dealing with a small group of voters, the preference profile containing the intended preferences and the one containing the erroneous preferences should be – if not identical – close to each other. Now, a plausible desideratum for a voting procedure is that mistakes of small voter groups and the accompanying small changes in preference profiles should not result in large changes in ensuing voting outcomes. In particular, the changes in the latter should not be larger as a result of mistaken reports of small voter groups than as a result of mistakes of larger ones. This is intuitively what voting power is about: changing the ballots of big groups should make a larger difference in voting outcomes than changing the ballots of small groups. This *prima facie* plausible desideratum turns, however, out to be incompatible with other intuitively compelling requirements of social choices.

The fundamental results in this area is due to Baigent (1987). To illustrate one of them, consider a drastic simplification of NATO and its policy options with regard to the on-going uprising in Libya.[4] Let us assume that there are only two partners in NATO (1 and 2) and two alternatives: impose a no-fly zone in Libya (NFZ) and refrain from military interference (R) in Libya. To simplify things even further, assume that only strict preferences are possible, i.e both decision makers have a strictly preferred policy. Four profiles are now possible:

We denote the voters' rankings in various profiles by $P_{mi}$ where $m$ denotes the number of the profile and $i$ the voter. We consider two types of metrics: one is defined on pairs of rankings and the other on profiles. The former is denoted by $d_r$ and the latter by $d_P$. The two metrics are related

[4] The argument is a slight modification of Baigent's (1987, 163) illustration.

| $P_1$ | | $P_2$ | | $P_3$ | | $P_4$ | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| NFZ | NFZ | R | R | R | NFZ | NFZ | R |
| R | R | NFZ | NFZ | NFZ | R | R | NFZ |

Table 5: Four two-voter profiles

as follows:

$$d_P(P_m, P_j) = \sum_{i \in N} d_r(P_{mi}, P_{ji}).$$

In other words, the distance between two profiles is the sum of distances between the pairs of rankings of the first, second, *etc.* voters. No further assumptions on the metric has been made.

Take now two profiles, $P_1$ and $P_3$, from Table 5 and express their distance using metric $d_P$ as follows:

$$d_P(P_1, P_3) = d_r(P_{11}, P_{31}) + d_r(P_{12}, P_{32}).$$

Since, $P_{12} = P_{32} =$ NFZ $\succ$R, and hence the latter summand equals zero, this reduces to:

$$d_P(P_1, P_3) = d_r(P_{11}, P_{31}) = d_r((NFZ \succ R), (R \succ NFZ)).$$

Taking now the distance between $P_3$ and $P_4$, we get:

$$d_P(P_3, P_4) = d_r(P_{31}, P_{41}) + d_r(P_{32}, P_{42}).$$

Both summands are equal since by definition:

$$d_r((R \succ NFZ), (NFZ \succ R)) =$$

$$d_r((NFZ \succ R), (R \succ NFZ)).$$

Thus,

$$d_P(P_3, P_4) = 2 \times d_r((NFZ \succ R), (R \succ NFZ)).$$

In terms of $d_P$, then, $P_3$ is closer to $P_1$ than to $P_4$. This makes sense intuitively.

The proximity of the social choices emerging out of various profiles depends on the choice procedures, denoted by $g$, being applied. Let us make two very mild restrictions on choice procedures, *viz.* that they are anonymous and respect unanimity. The former states that the choices are not dependent on the labelling of the voters. The latter, in turn, means that if all voters agree on a preference ranking, then that ranking is chosen. In our example, anonymity requires that whatever is the choice in $P_3$ is also the choice in $P_4$ since these two profiles can be reduced to each other by relabelling the voters. Unanimity, in turn, requires that $g(P_1) = NFZ$, while $g(P_2) = R$. Therefore, either $g(P_3) \neq g(P_1)$ or $g(P_3) \neq g(P_2)$. Assume the former. It then follows that $d_r(g(P_3), g(P_1)) > 0$. Recalling the implication of anonymity, we now have:

$$d_r(g(P_3), g(P_1)) > 0 = d_r(g(P_3, g(P_4)).$$

In other words, even though $P_3$ is closer to $P_1$ than to $P_4$, the choice made in $P_3$ is closer to - indeed identical with - that made in $P_4$. This argument rests on the assumption that $g(P_3) \neq g(P_1)$. Similar argument can, however, be made for the alternative assumption, *viz.* that $g(P_3) \neq g(P_2)$. The example, thus, shows that anonymity and respect for unanimity cannot be reconciled with a property called proximity preservation (Baigent 1987; Baigent and Klamler 2004): choices made in profiles more close to each other ought to be closer to each other than those made in profiles less close to each other.

The example shows that small mistakes or errors made by voters are not necessarily accompanied with small changes in voting outcomes. Indeed, if the true preferences of voters are those of $P_3$, then voter 1's mistaken report of his preferences leads to profile $P_1$, while both voters' making a mistake leads to $P_4$. Yet, the outcome ensuing from $P_1$ is further away from the outcome resulting from $P_3$ than the outcome that would have resulted had more - indeed both - voters made a mistake (whereupon $P_4$ would have emerged). This example shows that voter mistakes do make a difference. It should be emphasized that the violation of proximity preservation occurs in a wide variety of voting systems, viz. those that satisfy anonymity and unanimity. This result is not dependent on any particular metric with respect to which the distances between profiles and outcomes are measured. Hence, it applies to all preference-based voting systems.[5] Expressed in another way the result states that in nearly all reasonable voting systems it is possible that a small group of voters has a greater impact on voting outcomes than a big group. Thus, we have yet another way of violating local monotonicity.

[5] A referee correctly points out that we are not requiring that the larger group has identical preferences. Instead their preference changes cancel out each other. Indeed, we have here an instance reversal bias discussed at some length by Nurmi (2005). The point, however, is that a small group of voters may move the outcome a longer distance than a large – albeit heterogenous – group under specific preference configurations.

| *issue* | *issue 1* | *issue 2* | *issue 3* | *majority alternative* |
|---|---|---|---|---|
| *criterion A* | X | X | Y | X |
| *criterion B* | X | Y | X | X |
| *criterion C* | Y | X | X | X |
| *criterion D* | Y | Y | Y | Y |
| *criterion E* | Y | Y | Y | Y |

Table 6: Ostrogorski's Paradox

# 6 The ambiguity of closeness

Preference-based power measures equate an actor's power with the closeness of the decision outcomes to his ideal point in a policy space.[6] In a single-dimensional policy space, this is a relatively straight-forward matter to determine, but in multidimensional spaces closeness of two points depends on the metric used. With different metrics one may end up with different order of closeness of various points to one's ideal point. But even in cases where the metric is agreed upon, we may encounter difficulty in determining which of two points is closer to an actor's ideal point. Ostrogorski's paradox (Table 6) illustrates this (Rae and Daudt 1976).

There are two decision alternatives, X and Y. An individual decision maker has to choose between them on the basis of information regarding their distance from the individual's ideal point on three issues, $1-3$. Table 6 indicates which alternative is closer to the voter's ideal point on each criterion and in each issue.[7]

If all issues and criteria are equally important to the individual, it is reasonable to assume that on each criterion the individual prefers that alternative that is closer to his ideal point on more issues than the other alternative. The right-most column indicates these preferred alternatives on each criterion. Under the above assumption of equal importance of criteria and issues, one would expect the individual to choose X rather than Y since X is preferred on three criteria out of five.

However, looking at Table 6 from another angle, it becomes evident that Y should be chosen since on every issue it is the alternative that is closer to the individual's ideal point on a majority of criteria. In other words, there are reasonable grounds for arguing that X is closer to the individual's ideal point than Y, but there are equally strong reasons to make the opposite claim.

[6] This section is based on Nurmi (2010).

[7] X and Y could be applicants for a job or candidates for a political office. The issues, in turn, could be any three important aspects of the office, e.g. foreign policy, financial policy and education policy. The criteria could be work experience, relevant linguistic skills, relevant formal education, relevant social network and relevant social skills.

Ostrogorski's paradox is one of a larger family of aggregation paradoxes. These play an important role in the social sciences in general and in spatial models in particular. They have, however, less dramatic role in preference-based power indices, since these typically assume away the problem exhibited by the paradox. To wit, it is assumed that the distance measurements are unambiguous – a relatively straight-forward assumption in single-dimensional models – i.e. their approach is to find out power relationships *assuming* that the voters measure distances between alternatives in a given manner. For our purposes Ostrogorski-type paradoxes, however, suggest another overlooked aspect in power studies, viz. the packaging of issues or criteria. This is clearly one facet of the agenda-control problematique that we touched upon earlier. By aggregating or dis-aggregating issues one may change the ordering of alternatives when their closeness determines the choice.

## 7 The proper setting for power indices

The challenges of *a priori* voting power indices are mostly related to settings involving more than two alternatives. In decision making involving two alternatives they are still useful tools in assessing the implications of changes in decision rules or seat distributions. The *a priori* nature should, of course, be held in mind. The practical influence over outcomes may grossly deviate from the *a priori* index values due to the fact that coalitions tend to have different likelihoods of forming. Also the "nature" of various decision making bodies plays a role in using power indices. Felsenthal and Machover (1998) distinguish between I-power and P-power, while Laruelle and Valenciano (2008) introduce a useful distinction between bargaining and take-it-or-leave-it committees. With these distinctions these authors aim at delineating the conditions of the applicability of the indices. It is likely that further work along this line will follow. Above we argued that agenda-institutions and voting rules deserve attention as determinants of not only voting outcomes but also of the distribution influence among actors. In very general terms, majoritarian voting rules (e.g. amendment, Copeland and Dodgson) assign power to majorities, while positional ones (esp. Borda) assign relatively more power to minorities. When we enter the multiple-alternative environment and leave the simple game setting behind, many kinds of issues arise which always complicate and sometimes contradict the conclusions derived in the two-alternative settings. In the preceding an attempt has been made to examine some of these.

# 8 Appendix

The Shapley-Shubik index value of player $i$ is:

$$\phi_i = \Sigma_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!}[v(S) - v(S \setminus \{i\})].$$

Here $s$ denotes the number of members of coalition $S$ and $n!$ is defined as the product $n \cdot (n-1) \cdot (n-2) \cdot \ldots 2 \cdot 1$. The expression in square brackets differs from zero just in case $S$ is winning but $S \setminus \{i\}$ is not. In this case, then, $i$ is a decisive member in $S$. In other words, $i$ has a swing in $S$. Indeed, the Shapley-Shubik index value of $i$ indicates the expected share of $i$'s swings in all swings assuming that coalitions are formed sequentially.

Player $i$'s PGI value $H_i$ is computed as follows:

$$H_i = \frac{\Sigma_{S* \subseteq N}[v(S*) - v(S * \setminus \{i\})]}{\Sigma_{j \in N} \Sigma_{S* \subseteq N}[v(S*) - v(S * \setminus \{j\})]}.$$

Here $S*$ is a minimal winning coalition, *i.e.* every proper subset of $S*$ is a losing coalition.

The Deegan-Packel-index value of player $i$, denoted $DP_i$, in turn, is obtained as follows:

$$DP_i = \frac{\Sigma_{S* \subseteq N} 1/s[v(S*) - v(S * \setminus \{i\})]}{\Sigma_{j \in N} \Sigma_{S* \subseteq N} 1/s[v(S*) - v(S * \setminus \{j\})]}.$$

The standardized Banzhaf index value of $i$ is defined as:

$$\bar{\beta}_i = \frac{\Sigma_{S \subseteq N}[v(S) - v(S \setminus \{i\})]}{\Sigma_{j \in N} \Sigma_{S \subseteq N}[v(S) - v(S \setminus \{j\})]}.$$

The absolute Penrose-Banzhaf index (Penrose 1946; Banzhaf 1965), in turn, is defined as:

$$\beta_i = \frac{\Sigma_{S \subseteq N}[v(S) - v(S \setminus \{i\})]}{2^{n-1}}.$$

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