Pathology or Revelation? - The Public Good index

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Abstract

This paper sets out from a discussion of the well-known fact that the PGI violates the axiom of local monotonicity (LM). It argues that cases of nonmonotonicity indicate properties of the underlying decision situations which cannot be brought to light by the more popular power measures, i.e., the Banzhaf index and the Shapley-Shubik index, that satisfy LM. The discussion proposes that we can constrain the set of games such that LM also holds for the PGI. A discussion of causality follows. It suggests that the nonmonotonicity can be the result of framing the decision problem in a particular way and perhaps even ask the wrong question. Correspondingly, the PGI can be interpreted as an indicator. The probabilistic relationship of Banzhaf index and PGI identifies the factor which is responsible for the formal difference between the two measures and therefore for the violation of LM that characterizes the PGI, but not the Banzhaf.

1 The PGI introduction

This paper focuses on the representation of causality in collective decision making by means of power and power measures and discusses the question whether the Public Good Index (PGI) is a suitable instrument for this representation. To answer this question we relate the PGI and, alternatively, the Banzhaf index to the NESS concept of causality. However, it shows that the answer also depends on whether we interpret the PGI as measure or as an indicator.

Section 2 discusses the well-known fact that the PGI violates the axiom of local monotonicity (LM), i.e., it is not guaranteed that a player that can make a larger contribution to winning than another has at least as large an index value. In section 3, we argue that cases of nonmonotonicity indicate properties of the underlying decision situations which cannot be brought to light by the more popular power measures, i.e., the Banzhaf index and the Shapley-Shubik index, that satisfy LM. The discussion proposes that we can constrain the set of games representing decision situations such that LM also holds for the PGI. This might be a helpful instrument for the design of voting bodies. The discussion of causality in section 4 suggests that the nonmonotonicity can be the result of framing the decision problem in a particular way and can perhaps even ask the "wrong question". The core of this section is dedicated to connecting power and responsibility in the case of collective decision making and collective action, i.e., when the cause for an outcome cannot be directly assigned to a particular individual agent. Based on the discussion in the previous sections, section 5 points out that the PGI can be interpreted as an indicator and thus even serve as a valuable instrument in cases where there are serious doubts raised whether it can be applied as a measure. To conclude, section 6 looks into the probabilistic relationship of Banzhaf index and PGI as elaborated independently by Widgrn (2002) and Brueckner (2002) that identifies the factor which is responsible for the formal difference between the two measures. Can we interpret this factor as the cause for the violation of LM that characterizes the PGI, but not the Banzhaf? Can we see from the properties of this factor whether the PGI will indicate a violation for a particular game, or not? However, these are questions that have not been answered as yet.

The normalized Banzhaf index of player i counts the number of coalitions S that have i as a swing player such that S is a winning coalition and $S \setminus \{i\}$ is a losing coalition for all $S \subset N$ if N is the set of all players of game v. For normalization this number is divided by the total number of swing positions that characterize the game v.

The PGI differs from the Banzhaf index inasmuch as only minimum winning coalitions (MWCs) are considered. S is a MWC coalition if $S \setminus \{i\}$ is a losing one, for all i (S, i.e., all players of a MWC coalition have a swing position. The PGI of player i, h_i, counts the number of MWCs that have i as a member and divides this sum by the sum of all swing positions the players have in all MWCs of the game. If m_i is the number of MWCs that have i as a member then is PGI value is

$$h_i = \frac{m_i}{\sum_{i \in N} m_i} \qquad (1)$$

The corresponding definition of the normalized Banzhaf index is

$$\beta_i = \frac{c_i}{\sum_{i \in N} c_i} \tag{2}$$

In (2), c_i is number of winning coalitions that have i as a swing player. The following analysis is based on these two power measures.

2 The pathology

In 1978, when Holler first applied the PGI to the study of a voting power distribution in a parliament, he concluded that facing the violation of LM "causes doubt" concerning the validity of this measure. Obviously, he found the "index of Banzhaf-Coleman type" which he used as an alternative "more adequate in the context of this analysis" (Holler 1978: 33). However, Holler (1982) argued that taking into consideration coalition formation, collective decision making and the public goods problem the focus on MWCs and thus on the PGI seems to be an adequate solution to measuring the distribution of voting power in the decision making body. This view was supported by the axiomatization of the PGI in Holler and Packel (1983) and Napel (1999). However, in their article "Postulates and Paradoxes of Relative Voting Power - A Critical Review", Dan Felsenthal and Mosh Machover (1995: 211) write that "it seems intuitively obvious that if $w_i \leq w_j$ then every voter j has at least as much voting power as voter i, because any contribution that i can make to the passage of a resolution can be equalled or bettered by j." They conclude that "any reasonable power index" should be required to satisfy local monotonicity, i.e., LM. Even more distinctly, they argue that any a priori measure of power that violates LM is 'pathological' and should be disqualified as a valid yardstick for measuring power (Felsenthal and Machover 1998: 221ff). This argument has been repeated again and again when it comes to the evaluation (and application) of the PGI and the Deegan-Packel index.[1]

A notorious example to illustrate the nonmonotonicity of the PGI is the voting game $v^0 = (51; 35, 20, 15, 15, 15)$. The corresponding PGI is

$$h(v^0) = (\frac{4}{15}, \frac{2}{15}, \frac{3}{15}, \frac{3}{15}, \frac{3}{15}),$$

indicating a violation of LM in the resulting distribution of a priori voting power.[2]

[1]The Deegan-Packel index was introduced in Deegan and Packel (1979). This measure considers the value of coalition to be a private good that is equally shared among the members of a coalition. For a recent discussion of this measure, taking a priori unions into account, see Alonso-Meijide et al. (2011).

[2]The corresponding Deegan-Packel index, $\rho(v^0) = (18/60, 9/60, 11/60, 11/60, 11/60)$, also shows a violation of LM.

The application of power indices is motivated by the widely shared "hypothesis" that the vote distribution is a poor proxy for a prior voting power. If this is the case, does it make sense to evaluate a power measure by means of a property that refers to the vote distribution as suggested by LM? Of course, our intuition supports LM. However, if we could trust our intuition, do we need the highly sophisticated power measures at all?[3]

3 The revelation

It has been argued that a larger voter j can be less welcome to join a (nonwinning) proto-coalition than a smaller voter i.[4] The intuitive argument is the following. Let's assume a voting game $v* = (51; 45, 20, 20, 15)$ and players 2 and 3 form a proto-coalition $S = \{2, 3\}$. The losing coalition S can be "transformed" into a winning coalition if either player 1 or player 4 or both join S. However, if player 1 joins, either individually or together with 4, then neither player 2 nor player 3 is critical to the winning of a majority, i.e., in the coalition $\{1, 2, 3\}$ neither 2 nor 3 is a swinger. If voting power refers to a swing position – and this is, with some modification, the kernel of all standard power measures – and players are interested in power, then it seems likely that players 2 and 3 prefer the "smaller" voter 4 to join S to form a winning coalition. This story tells us that it could well be that a larger player is not always welcome to form a winning coalition if a smaller one does the same job. But does this mean that only minimum winning coalitions will form? Empirical evidence speaks against this conclusion. However, it has been repeatedly argued that if (nonminimal) winning coalitions with surplus players form then this is due to luck or ideology (i.e. preferences) and should not be taken into consideration when it comes to represent a priori voting power.[5] But there are perhaps more straightforward arguments in favor of MWCs and the application of the PGI.

In Holler and Napel (2004a, 2004b) it has been argued that the PGI shows nonmonotonicity with respect to the vote distribution (and thus confirms that the measure does not satisfy LM) if the game is not decisive, as the above weighted voting game $v^0 = (51; 35, 20, 15, 15, 15)$, or improper (for an example, see section 4 below) and therefore indicates that perhaps we should worry about the design of the decision situation. The more popular power measures, i.e., the Shapley-Shubik index or the Banzhaf index satisfy LM and thus do not indicate any particularity if the game is neither decisive nor proper. Interestingly, these measures also show a violation of LM if we consider a priori unions and the equal probability of permutations and coali-

[3] See Holler (1997) and Holler and Nurmi (2010) for this argument.

[4] For a discussion of coalition formation, see e.g. Hardin (1976), Hart and Kurz (1984), Holler (2011), Miller (1984), and Riker (1962).

[5] See Holler (1982) and Holler and Packel (1983). See also Widgrén (2002).

tions, respectively, does no longer apply.[6] This suggests that a deviation of the equal probability of coalitions causes a violation of LM.

The concept of a priori unions or pre-coalition is rather crude when applied to the PGI as the PGI implies that certain coalitions will not be taken into consideration at all, i.e., have a probability of zero of forming. Note since the PGI considers MWC only, this is formally equivalent to put a zero weight on coalitions that have surplus players. Is this the ("technical") reason why the PGI may show nonmonotonicity? We will come back to this question in section 6 below.

Instead of accepting the violation of LM, we may ask which decision situations guarantee monotonic results for the PGI. An answer to this question may help to design adequate voting bodies. Obviously, the PGI satisfies LM for unanimity games, dictator games and symmetric games. The latter are games that give equal power to each voter; in fact, unanimity games are a subset of symmetric games. Note that for these types of games the PGI is identical with the normalized Banzhaf index.

In Holler et al. (2001), the authors analyze alternative constraints on the number of players and other properties of the decision situations. For example, it is obvious that local monotonicity will not be violated by any of the known power measures, including PGI, if there are n voters and $n-2$ voters are dummies. It is, however, less obvious that local monotonicity is also satisfied for the PGI if one constrains the set of games so that there are only $n-4$ dummies. A hypothesis that needs further research is that the PGI does not show nonmonotonicity if the voting game is decisive and proper and the number of decision makers is smaller than 6. (Perhaps this result also holds for a larger number of decision makers but we do not know of any proof.) The idea of restricting the set of games such that LM applies for PGI has been further elaborated in Alonso-Meijide and Holler (2009) in the form of "weighted monotonicity of power." It seems that these considerations are relevant for all power indices if we drop the equal probability assumption and, for example, take the possibility of a priori unions into account.

4 Causality and power

The elaboration of various power measures and their discussion is meant to increase our understanding of power in collectivities and also to be of help in the design of voting bodies. A relatively new application of these measures results from their formal equivalence with representations of causality in collective decision making. Given this, it seems a short step to equate power and responsibility.

[6] See Alonso-Meijide and Bowles (2005) for examples of voting games with a priori unions and Alonso-Meijide and Holler (2009), Alonso-Meijide et al. (2009) as well as Holler and Nurmi (2010) for a discussion.

The specification of causality in the case of collective decision making with respect to individual agents cannot be derived from the action and the result as both are determined by the collectivity. They have to be traced back to decision making and, in general, the decision making process. However, collective decision making has a quality that substantially differs from individual decision making. For instance, an agent may support his favored alternative by voting for another alternative or by not voting at all. Nurmi (1999, 2006) contain a collection of such "paradoxes". These paradoxes tell us that we cannot derive the contribution of an individual to a particular collective action from the individual's voting behavior.[7] Trivially, a vote is not a contribution, but a decision. Resources such as power, money, etc. are potential contributions and causality might be traced back to them if a collective action results. As a consequence causality follows even from votes that do not support the collective action. This is reflected by everyday language that simply states that the Parliament has decided when in fact a decision was made by a majority smaller than 100 per cent. But how can we allocate causality if it is not derived from decisions?

Alternatively, we may assume in what follows that the vote (even in committees) is secret and we do not know who voted "yes" or "no". Moreover, in general, there are more than two alternatives and the fact that a voter votes "yes" for A in a last pairwise voting only means that he/she prefers A to B or does not want to abstain, but this vote does not tell us why and how alternatives C, D, etc. were excluded.[8] Thus, an adequate concept of causality (and responsibility) does not presuppose a voting result that is known and indicates who said "yes" and who said "no".[9]

For an illustration, imagine a five-person committee $N = \{1, 2, 3, 4, 5\}$ that makes a choice between the two alternatives x und y.[10] The voting rule specifies that x is chosen if either (i) 1 votes for x, or (ii) at least three of the players $2-5$ vote for x. Let's assume all individuals vote for x. What can be said about causality? Clearly, this is a case of over-determination and the allocation of causation is not straightforward. Alternatively, we may assume that all we get to know is that x is decided, but we do not know who voted for or against it. In both cases, we may conclude on causality by looking at possible winning coalitions. For example, the action of agent 1 is a member of only one minimally sufficient coalition, i.e. decisive set, while the actions of each of the other four members are in three decisive sets each. If we take the membership in decisive sets as a proxy for causation, and standardize

[7]For a discussion and examples, see Nurmi (2010) Holler and Nurmi (2011).

[8]See "The Fatal Vote: Berlin versus Bonn" (Leininger 1993) for an illustration.

[9]See Braham (2005, 2008), Braham and Holler (2009), and Holler (2007) for this treatment of causality. It differs from the approach discussed in Felsenthal and Machover (2009) which refers to particular (voting) *results*, and not for the *potential* of having contributed to it.

[10]The rest of this section derives from Holler (2011b).

such that the shares of causation adds up to one, then vector

$$h^0 = (\frac{1}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13})$$

represents the degrees of causation. Braham and van Hees (2009: 334), who introduced and discussed the above case, conclude that "this is a questionable allocation of causality." They add that "by focusing on minimally sufficient conditions, the measure ignores the fact that anything that players $2-5$ can do to achieve x, player 1 can do, and in fact more - he can do it alone." We share this specification, but does it apply in collective decision making?

Let's review the above example. Imagine that x stands for polluting a lake. Now the lake is polluted, and all five members of N are under suspicion of having contributed to its pollution. Then h^0 implies that the share of causation for 1 is significantly smaller than the shares of causation of each of the other four members of N. If responsibility and perhaps even punishment follow from causation then the allocation h^0 seems highly pathological. As a consequence Braham and van Hees propose to apply the weak NESS instead of the strong one, i.e., not to refer to decisive sets, but to consider sufficient sets instead and count how often an element i of N is a necessary element of a sufficient set (i.e., a NESS). Taking care of an adequate standardization so that the shares add up to 1, we get the following allocation of causation:

$$\beta^0 = (\frac{11}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23}).$$

The result expressed by b^0 looks much more convincing than the result proposed by h^0, doesn't it? Note that the b-measure and h-measure correspond to the Banzhaf index and the PGI, respectively, and can be calculated accordingly.

So far the numerical results propose the weak NESS test and thus the application of the normalized Banzhaf index. However, what happened to alternative y? If y represents "no pollution" then the set of decisive sets consists of all subsets of N that are formed of the actions of agent 1 and the actions of two out of agents $2, 3, 4$ and 5. Thus, the actions of 1 are members of six decisive sets while the actions of $2, 3, 4$ and 5 are members of three decisive sets each. The corresponding shares are given by the vector

$$h* = (\frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$$

Obviously, $h*$ looks much more convincing than h^0 and the critical interpretation of Braham and van Hees does no longer apply: agent 1 cannot bring about y on its own, but can cooperate with six different pairs of two other agents to achieve this goal.

Note that the actions (votes) bringing about x represent an improper game – two (disjunct) "winning" subsets can exist at the same time[11] – while the determination of y can be described by a proper game. However, if there are only two alternatives x and y then "not x" necessarily implies y, irrespective of whether the (social) result is determined by voting or by polluting. The h-values indicate that it seems to matter what issue we analyze and what questions we raise while the Banzhaf index with respect to y is identical to the one for $x : \beta^0 = b*$.

Already Coleman (1971) developed a measure of power of a member to initiate action and a measure of power of a member to prevent action. However, Brams and Affuso (1967) demonstrated that, submitted to adequate linear transformations, the two measures yield the normalized Banzhaf index. This is not the case with public good values h^0 and $h*$ above.

5 Measure or indicator?

Whether we should apply h and β, or a third alternative, to measure causation seems an open question, and this paper will not give an answer to this question. To conclude, the PGI and thus the strong NESS concept may produce results that are counter-intuitive at first glance. However, in some decision situations they seem to reveal more about the power structure and corresponding causality allocation than the Banzhaf index and the corresponding weak NESS concept. However, if we want to relate responsibility to power then the nonmonotonicity, i.e. the violation of LM, that represents the strong NESS test of the PGI is quite a challenge: If the collective choice is made through voting then it is not guaranteed that a voter with a larger share of votes has at least as much responsibility for the collectively determined outcome as a voter with a smaller share. From the example above we can learn that nonmontonicity might indicate that we asked the wrong question: Is the responsibility with respect to keeping the lake clean or is it with polluting the lake? Both alternatives may imply the sharing of the costs of cleaning it. Of course, there is no quantitative answer to this question, but the quantification by the index showed us that there might be a problem with the specification of the game model.

A possible answer of whether the PGI represents a pathology or not, might be found in this quality-quantity duality: the use of quantity measures to indicate qualitative properties of (voting) games. Whether a game is improper or non-decisive is not a matter of degree. Indicators show red lights or make strange noises when an event happens that has some meaning in a particular context. This does not necessarily mean that the corresponding indicator functions as a measure, but often it does and when it does it summarizes the measured values in the form of signals. What is a relevant

[11] Player 1 is a dictator in guaranteeing x, but x can also be achieved without his support.

and an appropriate signal of course depends on the context and the recipient. Red lights are not very helpful for blind people. What are the relevant and appropriate signals that correspond to power measures? What are the problems that should be uncovered and perhaps even be solved? What are the properties a power measure has to satisfy when it should serve as a signal? These are questions that we cannot answer in a systematic way without reference to a particular issue.

6 On the relationship of Banzhaf index and PGI

Widgrén (2002) proved the following linear relationship that relates the normalized Banzhaf index (β_i) and the PGI (h_i).[12]

$$\beta_i = (1-\pi)h_i + \pi\varepsilon_i \tag{3}$$

where
$\varepsilon_i = \frac{\overline{c_i}}{\sum_{i\in N}\overline{c_i}}$ and $\pi = \frac{\sum_{i\in N}\overline{c_i}}{\sum_{i\in N}c_i}$
Here, c_i represents the number of (crucial) coalitions that contain player i as a swing player and $\overline{c}_i$ represents the number of coalitions which have a swing player i, but are *not* minimum winning. If we apply equation (3) to the voting game x discussed in section 4 and the corresponding power indices β^0 and h^0, then we have $c_1 = 11, c_2 = c_3 = c_4 = c_5 = 3, \bar{c_1} = 10$ and $\bar{c_i} = 0$ for $i = 2,3,4,5$. As a consequence, $\pi = \frac{10}{23}, \varepsilon_1 = \frac{10}{10}$, and $\varepsilon_i = 0, i = 2,3,4,5$. It is easy to check that these values are consistent with equation (3) and the values of β^0 and h^0.

Loosely speaking, the coalitions represented by are the source of the difference between the normalized Banzhaf index, $beta_i$, and the PGI, h_i. Can we identify the corresponding factors in (3) as the cause for the violation of LM that characterizes the PGI, but not the Banzhaf? Can we see from the properties of this factor whether the PGI will indicate a violation for a particular game, or not? - These questions have not been answered so far, but it is immediate from (3) that the PGI satisfies LM for unanimity games, dictator games and symmetric games. For these games $\pi = 0$ and the PGI equals the normalized Banzhaf index (which satisfies LM for all voting games).

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[12] Widgrén uses the symbols θ_i for the PGI and C_i for the set of crucial coalitions that contain i as a swing player. Correspondingly, c_i is the number of elements of C_i.

References

Alonso-Meijide, J.M. and C. Bowles (2005), Power indices restricted by a priori unions can be easily computed and are useful: A generating function-based application to the IMF, *Annals of Operations Research 137*, 21-44.

Alonso-Meijide, José Maria, Balbina Casas-Mendez, Manfred J. Holler and Silvia Lorenzo-Freire (2008), Computing power indices: Multilinear extensions and new characterizations", *European Journal of Operational Research 188*, 540-554.

Alonso-Meijide, José Mara and Manfred J. Holler (2009), Freedom of choice and weighted monotonicity of power, *Metroeconomica 60 (4)*, 571-583.

Alonso-Meijide, José Mara, Balbina Casas-Mendez, Gloria Fiestas-Janeiro, Manfred J. Holler, and Andreas Nohn (2010), Axiomatizations of public good indices with a priori unions, *Social Choice and Welfare 35*, 517-533.

Alonso-Meijide, José Maria, Balbina Casas-Mendez, Gloria Fiestras-Janeiro and Manfred J. Holler (2011), The Deegan-Packel index for simple games with a priori unions, *Quality and Quantity 45 (2)*, 425-439.

Alonso-Meijide, José Mara, Carlos Bowles, Manfred J. Holler and Stefan Napel (2009), Monotonicity of power in games with a priori unions, *Theory and Decision 66*, 17-37.

Brams, Steven S. and Paul J. Affuso (1976), Power and size: A new paradox, *Theory and Decision 7*, 29-56.

Braham, M. (2005), Causation and the measurement of power, in: G. Gambarelli and M. J. Holler (eds.), *Power Measures III* (*Homo Oeconomicus 22*), Munich: Accedo-Verlag, 645-553.

Braham, M. (2008), Social power and social causation: Towards a formal synthesis", in: M. Braham and F. Steffen (eds.), *Power, Freedom, and Voting: Essays in Honour of Manfred J. Holler*, Berlin and Heidelberg: Springer, 1-21.

Braham, M. and van Hees, M (2009), Degrees of causation, *Erkenntnis 71*, 323-344.

Braham, M. and M.J. Holler (2009), Distributing causal responsibility in collectivities, in: R. Gekker and T. Boylan (eds.), *Economics, Rational Choice and Normative Philosophy*, London and New York: Routledge, 145-163.

Brueckner, Matthias (2002), Extended probabilistic characterization of power indices, in M. J. Holler and G. Owen (eds.), *Power Measures, Volume 2, Homo Oeconomicus 19*, 287-398.

Coleman, James S. (1971), Control of collectivities and the power of a collectivity to act, in: B. Liebermann (ed.), *Social Choice*, New York: Gordon and Breach, 269-301.

Deegan, John Jr. and Edward W. Packel (1979), A new index of power for simple n-person games, *International Journal of Game Theory 7*, 113-123.

Felsenthal, D. and M. Machover (1995), Postulates and paradoxes of relative voting power - a critical appraisal, *Theory and Decision 38*, 195-229.

Felsenthal, D. and M. Machover (1998), *The Measurement of Voting Power. Theory and Practice, Problems and Paradoxes*, Cheltenham: Edward Elgar.

Felsenthal, Dan S. and Mosh Machover (2009), Necessary and sufficient conditions to make the numbers count, *Homo Oeconomicus 26*, 259-271.

Hardin, R. (1976), Hollow victory: The minimum winning coalition, *American Political Science Review 70*, 1202-1214.

Hart, S. and M. Kurz (1984), Stable coalition structures, in: M.J. Holler (ed.), *Coalitions and Collective Action*, Wuerzburg and Vienna: Physica-Verlag, 235-258.

Holler, Manfred J. (1978), A prior party power and government formation, *Munich Social Science Review 4*, 25-41. (Republished in M.J. Holler (ed.), *Power, Voting, and Voting Power*, Wrzburg and Vienna: Physica-Verlag, 1982, 273-282.)

Holler, Manfred J. (1982), Forming coalitions and measuring voting power, *Political Studies 30*, 262-271.

Holler, M.J. (1985), Strict proportional power in voting bodies. *Theory and Decision 19*, 249-258.

Holler, Manfred J. (1997), Power, monotonicity and expectations, *Control and Cybernetics 26*, 605-607.

Holler, Manfred J. (2007), Freedom of choice, power, and the responsibility of decision makers, in: J.-M. Josselin and A. Marciano (eds.), *Democracy, Freedom and Coercion: A Law and Economics Approach*, Cheltenham: Edward Elgar, 22-45.

Holler, Manfred J. (2011a), Coalition Theory, in: K. Dowding (ed.), *Encyclopedia of Power*, London: Sage Publications.

Holler, Manfred J. (2011b), EU decision-making and the allocation of responsibility, Prepared for the Research Handbook on the Economics of European Union Law, edited T. Eger and H.-B. Schfer.

Holler, M.J. and S. Napel (2004a), Local monotonicity of power: Axiom or just a property, *Quality and Quantity 38*, 637-647.

Holler, M.J. and S. Napel (2004b), Monotonicity of power and power measures, *Theory and Decision 56*, 93-111.

Holler, Manfred J. and Andreas Nohn (2009), The public good index with threats in a priori unions, *Essays in Honor of Hannu Nurmi (Homo Oeconomicus 26)*, 393-401.

Holler, Manfred J. and Hannu Nurmi (2010), Measurement of power, probabilities, and alternative models of man, *Quality and Quantity 44*, 833-847.

Holler, Manfred J. and Hannu Nurmi (2011), Aspects of Power Overlooked by Power Indices, manuscript.

Holler, Manfred J. and Edward W. Packel (1983), Power, luck, and the right index", *Journal of Economics (Zeitschrift fr Nationalkonomie) 43*, 21-29.

Holler, M.J., R. Ono, and F. Steffen (2001), Constrained monotonicity and the measurement of power, *Theory and Decision, 50*, 385-397

Holler, Manfred J. and Mika Widgren (1999), The value of a coalition is power, *Homo Oeconomicus 15*, 485-501 (Reprinted in *Notizie di Politeia 59*, 2000, 14-28).

Leininger, Wolfgang (1993), The fatal vote: Berlin versus Bonn, *Finanzarchiv 50*, 1-20.

Miller, N.R. (1984), Coalition formation and political outcomes: A critical note, in: M.J. Holler (ed.), *Coalitions and Collective Action*, Wuerzburg and Vienna: Physica-Verlag, 259-265.

Napel, Stefan (1999), The Holler-Packel axiomatization of the Public Good Index completed, *Homo Oeconomicus 15*, 513-520.

Nurmi, Hannu (1999), *Voting Paradoxes and How to Deal with Them*, Berlin, Heidelberg, and New York: Springer.

Nurmi, Hannu (2006), *Models of Political Economy*, London and New York: Routledge.

Nurmi, H. (2010), On the relative unimportance of voting weights: Observations on agenda-based voting procedures," in M. Cichocki and K. Zyczkowski, (eds.), *Institutional Design and Voting Power in the European Union*, Farnham, Surrey: Ashgate, 171-180.

Riker, W. (1962), *The Theory of Political Coalitions*, New Haven and London: Yale University Press.

Widgrén, Mika (2002), On the probabilistic relationship between the public good index and the normalized Banzhaf index," in M. J. Holler and G. Owen (eds.), *Power Measures, Volume 2, Homo Oeconomicus 19*, 273-386.