# Niccolò Machiavelli (1469-1527)

# Power and Freedom of Choice,

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## 1. The Concept of Freedom of Choice

**X is the set of opportunities.**

**Z is the power set of X that has X, $\varnothing$ and**

**all A, B $\subseteq$ X as elements.**

***R* is the Freedom of Choice relation such that,**

**for all A, B $\in$ Z, A*R*B expresses: "The degree of freedom of choice of A is at least as large as the degree of freedom of B."**

*R* is a binary relation with respect to the opportunity sets in *Z*.

**Note that**

**- if A*R*B and B*R*A, then A*I*B:** "The degree of freedom of choice of A is **as** large as the degree of freedom of B" (equality relation of freedom of choice)

- if **A*R*B and *not* B*R*A, then A*P*B:** "The degree of freedom of choice of A is **larger than** the degree of freedom of B" (strict freedom of choice relation)

**In what follows we look at**

**- three specifications of R, i.e., $R_{\#}$, $R_{\alpha}$, and $R_{\subseteq}$,**

**- a power approach, and**

**- a causality interpretation.**

There other concepts camparing sets,

e.g., the **indirect utility approach**: identifying the value of a choice set with its best element.

## 2. Why should we be interested in *R*?

**(1) *R* could have an impact on our wellbeing in addition to the choice as such,** i.e., it could be an argument of our utility function. Is there a preference for freedom of choice?

**(2) *R* could be assumed to augment social welfare,** e.g., cultural diversity is considered an asset.

**(3) *R* could be of interest if preferences are not fixed (or given) when A, B $\subseteq$ X are offered.**

**(4) *R* could be of interest if A, B $\subseteq$ X are fuzzy and elements are not fully fixed.**

**(5) *R* could help to clarify power and causality - and corresponding axioms, properties, conditions, etc.**
**- Dewey's experience!**

**3. $R_{\#}$,** the c**ardinality measure of freedom of choice**

**Pattanaik and Xu (1990) propose three properties (axioms) that uniquely characterice a binary relation *R* on opportunity sets in Z.**

**Given x, y$\in$ X:**

Property 2.1 (**Simple Anonymity**).

**For all x, y in X, {x}*I*{y}.**

(expresses indifference between *no choice* situations)

Property 2.2 (**Simple Strict Monotonicity). For all distinct x, y $\in$ X, {x, y}*P*{x}**

(compares a choice and a *no choice* situation: Having a choice should "increase" freedom of choice)

Property 2.3 **(Simple Independence). For all A,B $\in$ Z and all x $\in$X\(A$\cup$B) follows [A*R*B $\Leftrightarrow$ A$\cup${x}*R*B$\cup${x}]**

(Is $\Leftrightarrow$ adequate? Element x has to "unrelated." )

Pattanaik and Xu (1990) proved that the only specification of *R* that satisfies these properties is the **counting relation** $R_{\#}$, i.e., $AR_{\#}B$ if $|A| \geq |B|$.

**Arguments against *R#***

(1) R# **ignores the preferences** of the individuals to whom the set of alternatives are allocated.

(2) R# **ignores complementary and substitutional relationships** among the alternatives.

(3) R# does not take into account of **differences in value (prices)** between elements of X.

(4) R# ignores the **capability** of the decision maker to make use of the various alternatives in X. (See Amartya Sen, 1985.)

**4. The α-ordering of Freedom of Choice**

**Marlies Klemisch-Ahlert** (1993) defines an ordering $R_\alpha$ such that

**for all A, B $\in$ Z,**

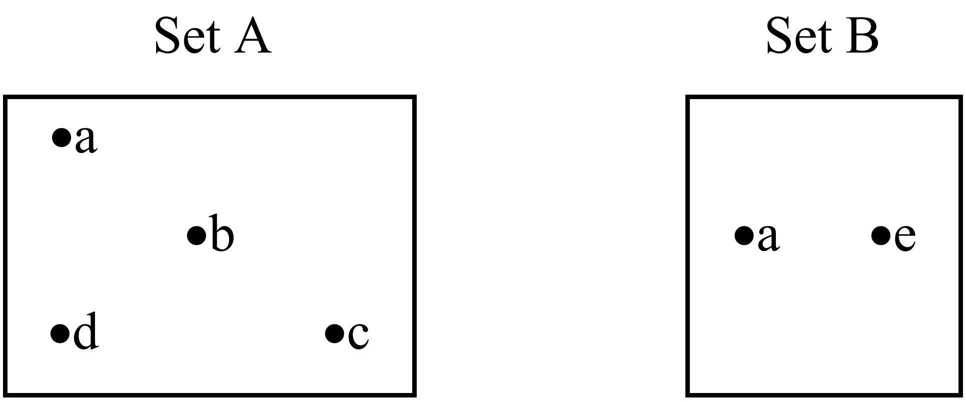
$$\textbf{(1)}\quad A R_\alpha B \Leftrightarrow \sum_{x \in A} \alpha(x) \geq \sum_{x \in B} \alpha(x)$$

**$\alpha$ is a mapping that assigns a weight $\alpha(x) > 0$ to every x in X.**

**More specifically $\alpha : X \rightarrow ]0,\infty[$.**

An ordering which satisfies (1) is called an **$\alpha$-ordering**.

Discuss:



The $\alpha$-ordering $R_\alpha$ satisfies *Properties 2.2 and 2.3.*, but does satisfy *Property 2.1* only if $\alpha(x) = \alpha(y)$, *which is in general not the case.*

**Klemisch-Ahlert (1993) demonstrated that**
**$R_\alpha$ satisfies Properties 2.2 and 2.3, and**

***Property 2.4*. For all x,y in X, $\{x\}R\{y\} \Leftrightarrow \alpha(x) \geq \alpha(y)$.**

However, a particular specification $R_\alpha$ is **not unique** in satisfying Properties 2.2, 2.3 and 2.4. There many $\alpha$-orderings that satisfy these properties and it could well be that A $R_{\alpha*}$B and B$R_{\alpha^\circ}$A for $\alpha^* \neq \alpha^\circ$.

## 5. The Inclusion Ordering $R_\subseteq$

**Definition 2.4. For all A, B $\in$ Z and B $\subseteq$ A $\Rightarrow$ A$R_\subseteq$B.**

**B $\subseteq$ A if B is a subset of A.**
**$R_\subseteq$ is the inclusion relation.**
**$R_\subseteq$ defines an incomplete ordering.**

**Marlies Klemisch-Ahlert** (1993) proved

**Lemma 2.1. The inclusion relation $R_{\subseteq}$ is the intersection of all α-orderings $R_\alpha$.**

**That is if B ⊆ A then** $\sum_{x \in A} \alpha(x) \geq \sum_{x \in B} \alpha(x)$

**and thus A$R_\alpha$B *irrespective of the α-weights,***
**given α:X → ]0,∞[.**

*Problem*: Why weights? x and y are of different social value! But if x is more important than y then, in fact, α(y) should be smaller than α(x): It is not "good" to have no freedom of choice on important issues.

**6. Power Relation π and the PGI**

**The power relation $i \pi j$ says that i is at least as powerful as j where i, $j \in N$, and**

**N is the set of agents,**

**$i \in N$ is an agent (a player) such that (1, …,i,…,n),**

**$S \subseteq N$ such that S is a coalition.**

Why and how to link freedom of choice to power.

1. In a social context, the **freedom of choice** of i is determined by the set of alternatives which i can guarantee himself "despite resistence".

2. The set of alternatives contains public goods, club goods, goods with substantial externalities, etc. What is the potential of a player i to **design** the public goods in his oportunity set? The answer is with the Public Good Index (PGI).

**Framework: A <u>social choice situation</u> *v* is given by the voting body (i.e. weighted voting game *v* = (d, w),**

**decision rule d = 51 and**

**distribution of voting weights**

**$w = (w_1, w_2, w_3, w_4, w_5) = (35, 20, 15, 15, 15)$.**

**Thus we have *v* = (51; 35, 20, 15, 15, 15).**

**N = {1,2,3,4,5} is the set of players**

**The set of minimum winning coalitions (MWC):**

**M(*v*) = {{1,2}, {1, 3, 4}, {1, 3, 5}, {1, 4, 5},**
**{2, 3, 4, 5}}**

**The corresponding Public Good Index (PGI) is:**

***h* = (4/15, 2/15, 3/15, 3/15, 3/15).**

We see that PGI violates local monotonicity.

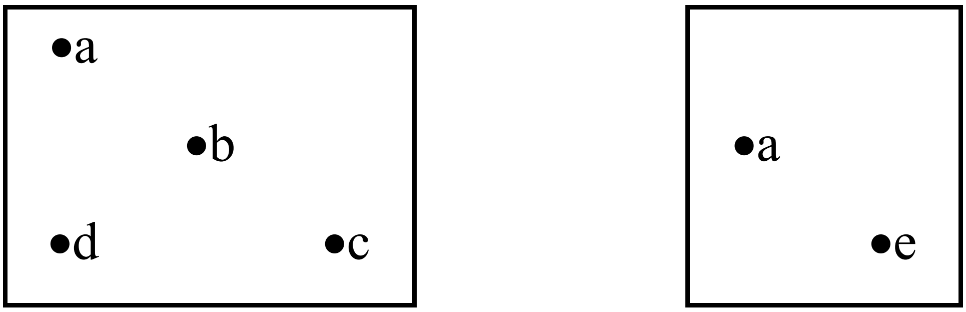
**Freedom of choice interpretation:**

**- Coalitions control the elements in X.**

**- The elements in X are collective goods.**

**- The degree of freedom of agent i is equivalent to i's power to determine the social outcome.**

**Controlled by 1** **Controlled by 2**




**The PGI corresponds to $R_{\#}$ freedom relation: counting.**

**Power indices do not take preferences into account.**

## 7. The $R_{\subseteq}$ - equivalence of PGI-ordering

**$M_i(u)$ is the set of minimum winning coalitions which have i as a member, $c_i(u)$ is the cardinality of $M_i(u)$ and $c(u)$ is the sum of these cardinalities over all players.**

$h_i(v) = \frac{c_i}{\sum_{i \in N} c_i}$ **defines the PGI of i.**

$c_i$ = number of **decisive sets** which have i as a member = $|M_i(v)|$, i.e., cardinality of $M_i(v)$.

We consider to games *u* and *v*, and define

**PGI-monotonicity.** Given $M_i(u) \supseteq M_i(v)$, a solution h on the set of all simple games satisfies PGI-monotonicity if, for any pair of simple games *u* and *v*,

$$h_i(u)c(u) \geq h_i(v)c(v) \qquad (1)$$

for all player $i \in N$ holds.

Alonso-Meijide, J.M., Casas-Méndez, B., Holler, M.J., and S.M. Lorenzo-Freire (2008) show that the PGI is the only index which satisfies PGI-monotonicity, symmetry, null player and efficiency.

1. Can we conclude from $M_i(u) \supseteq M_i(v)$ that i has at least as much **freedom of choice** in game $u$ than in game $v$?
2. Can we conclude from $M_i(u) \supseteq M_j(u)$ that i has at least as much **freedom of choice** in game $u$ than j?
3. Can we conclude from $h_i(u) \geq h_j(u)$ that i has at least as much **freedom of choice** in game $u$ than j?

If $h_i(u) > h_i(v)$, then, in accordance with $R_{\subseteq}$ , we conclude that game $u$ contains more freedom of choice than game $v$ for agent i if $M_i(u) \supset M_i(v)$ and $M(u) \subseteq M(v)$ hold.

"Playing" with $h_i(v) = \frac{c_i}{\sum_{i \in N} c_i}$

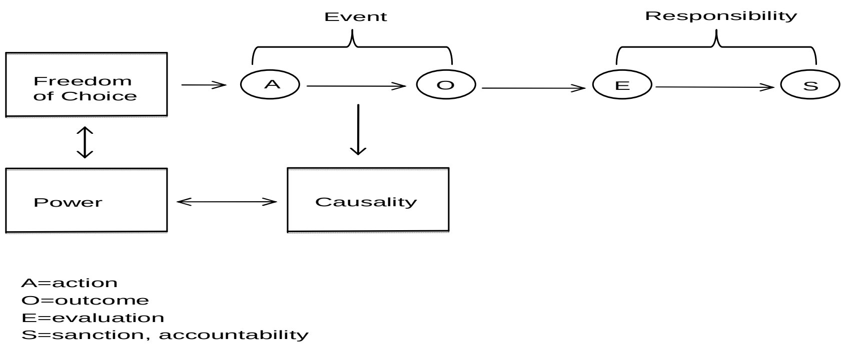
If $h_i(u) > h_i(v)$, then, in accordance with $R_{\#}$ , we conclude that game $u$ contains more freedom of choice than game $v$ for agent i if $c_i(u) > c_i(v)$ and $c(u) \leq c(v)$ hold.

Of course, $M(u) \subseteq M(v) \Rightarrow c(u) \leq c(v)$, but the reverse does not "necessarily" hold.

Cesare Borgia (1475-1507)

1491 Bishop
1492 Archbishop of Valencia
1493 Cardinal
1507 deadly wounded at Viana when fighting for his brother-in-law, King John of Navarra

**8. Causality and Responsibility**




**Given a collective decision problem *v* = (d, w). What is the responsibility of i with respect to the choices of *v*?**

**NESS concept: necessary element of a sufficient set – then the Banzhaf index expresses responsibility.**

This is called the **Weak NESS** test. (See Braham and van Hees, 2009.)

The normalized Banzhaf index is defined as

$$\beta_i(v) = \frac{\beta_i'(v)}{\sum_{i\in N}\beta_i'(v)} = \frac{\#\,\text{swings}(i)}{\sum_{i\in N}\#\,\text{swings}(i)}$$

**EDS concept: element of a decisive set – then PGI expresses responsibility. – Necessary elements: Minimum winning coalitions only**

This is called the **Strong NESS** test. (See Braham and van Hees, 2009.)

**Weak or strong causation?**

**9. An illustration:**

Given a five-person committee N = {1,2,3,4,5},

two alternatives: {x, y}.

*x* is chosen if either

(i) 1 votes for *x*, or

(ii) at least three of the players 2-5 vote for *x*.

Then, taking care of **decisive sets**, we get

$$h° = (\frac{1}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13})$$

Imagine that *x* stands for polluting a lake and the kake is polluted, is *h*° an adequate sharing rule of the costs for cleaning the lake?

Taking care of **sufficient sets**, we get

$$b = (\frac{11}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23})$$

*b* looks much more convincing than the result proposed by *h*°, doesn't? But what is about *y*.

If *y* represents "no pollution," then the set of decisive sets consists of all subsets of N formed of the actions of 1 and the actions of two agents out of {2, 3, 4, 5}.

Then $$h^* = (\frac{2}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$$

and $$b^* = (\frac{11}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23})$$

## Proper and nonproper games – does it matter?

Note that game v° is improper

…results might be over-dtermined.

**Conclusion**: The *h*-values indicate that it seems to matter what issue we analyze and what questions we raise while for the Banzhaf index we have $b° = b*$.

## 10. A case

… that **Ronald Dworkin** "uses several times involves a class of plaintiffs suing a number of pharmaceutical companies, who over a long period manufactured a drug the plaintiffs took during pregnancy (to prevent miscarriage).[1] The manufacturers do not deny that they continued manufacturing and advertising the drug after it became clear that it had carcinogenic side effects, but each company says that unless a particular plaintiff can actually prove that *its* batch of pills caused her cancer, she has no remedy against that particular company."

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[1]Sindell v. Abbott Laboratories, 20 Cal.3d 588,607 P.2d 924 (1980).

**There is a causality problem!?**

**Is this right**? Or may a court do what the California courts did and award damages against the companies on the basis of their proportionate share of the market **without proof of particular causality**?" (p.57f)[2]

**Questions**:

1. Were the drugs still sold after the judgement?
2. Were the drugs improved?
3. Is the market-share allocation fair and efficient?

**Rose-Ackerman, Susan** (1990), "Market-share allocations in tort law: Strengths and weaknesses," Journal of Legal Studies 19, pp. 739-746.

1. The level of damage payments (i.e. compensation) determined by marginal damages.
2. Quality is a "local public good." Improvement needs collective action or regulation.

**Using power indices instead of market shares to allocate responsibility - and damage payments?**

**…in case of collective decision making through voting, e.g., in the EU???**

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[2] Waldron, Jeremy (2006), "How Judges Should Judge", Review of Justice in Robes, by **Ronald Dworkin**, Belknap Press/Harvard University, New York Review of Books, 53(13) August 10, 54-59.

“A few years ago, I’d read *The Prince* and I liked it a lot. Much of what Machiavelli said made sense, but certain things stick out wrong – like when he offers the wisdom that it’s **better to be feared than loved**, it kind of makes you wonder if Machiavelli was thinking big. I know what he meant, but sometimes in life, **someone who is loved can inspire more fear than Machiavelli ever dreamed of.**” (Dylan, 2005, p.140f)

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